Acknowledgment

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Pulsating Flows about Axisymmetric Concave Bodies

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I. Introduction

SURFACE indentations in the nose region of axisymmetric blunt bodies (Fig. 1) in high-speed flow has been considered as a means of decreasing drag and heating in highspeed cruise or glide vehicles and increasing drag in planetary re-entry vehicles. If flow separation occurs, it is not always steady—it may be periodically unsteady. Two distinct modes of instability have been observed.

In the "pulsation" mode (Fig. 1a) the conical separation bubble formed on the concave part of the body periodically inflates and expands radially, taking a hemispherical shape. In the "oscillation" mode the conical foreshock, which envelops the separation bubble, and the accompanying shear layer oscillate laterally and their shape changes periodically from concave to convex (Fig. 1b). The pulsation mode was first observed by Mair 1 and the oscillation mode by Bogdonoff and Vas.² The terminology is due to Kabelitz.³

A comparison shows that the oscillating flow about a concave body is a subcase of the self-sustained oscillations of impinging free shear layers, reviewed by Rockwell and

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Naudascher. 4 Thus, the primary mechanism of the oscillation is related to the stability of the shear layer that envelops the conical separation bubble. It has been shown by the present author⁵ that critical parameters for the occurrence of oscillation are the volume of the axisymmetric cavity and the geometry of the shoulder of the afterbody where the shear layer reattaches.

However, the mechanism of the pulsation is quite different. It will be shown in this Note that the pulsation is due to the effect which an annular supersonic jet, appearing at the shock intersection of the foreshock and aftershock, has on the separation bubble.

II. The Mechanism of the Pulsation Mode

Examination of the Schlieren photographs presented in Fig. 2 indicates that at the starting phase of each cycle of pulsation, the flow conditions prevailing resemble an impulsive flow in which the shock envelope retains a position corresponding to an inviscid flowfield. The persistence of the shock intersection during this initial phase, in which the separation bubble inflates (Figs. 2a-2c), plus the existing pressure imbalance behind the two shocks caused us to analyze the flow in the vicinity of the shock intersection and thus to discover the mechanism of this explosive instability.

Applying a quasisteady analysis at the shock intersection, we discovered that the shock formation is similar to one of the series studied by Edney⁶ known to result in the production of a supersonic jet. This is schematically shown in Fig. 3, where the present unsteady case is compared with the Edney IV shock formation.

If the hypothesis of the existence of the supersonic jet is valid, the mechanism of the pulsation is as follows. For a certain fraction of the time of the cycle (which has been found experimentally 7 to be about one-tenth), the high-speed flow processed by the weak foreshock is channeled between the weak foreshock and the inflating separation bubble and is directed toward the body where it impinges as an annular supersonic jet. The separation bubble formed in the cavity of the body is thus inflated very rapidly, causing the distortion of the foreshock which forces the shock intersection away from the body and thus cutting off the source of the flow which causes the inflation. The high-pressure separated flow then expands and, since no disturbance remains to sustain the now strong foreshock, it collapses downstream and flows past the afterbody. Obviously, as the strong shock moves toward the afterbody, the forebody is exposed to the freestream, the weak shock reappears, and the initial conditions of the impulsive flow are re-established.

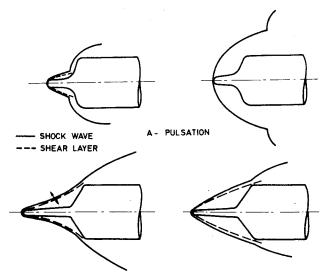
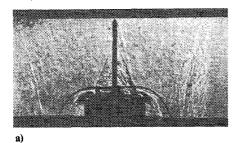
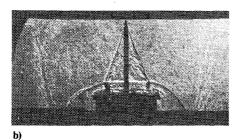
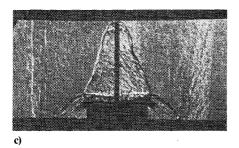
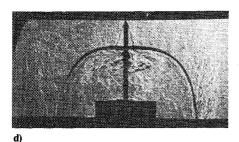


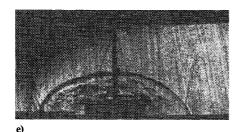
Fig. 1 Classification of instabilities.











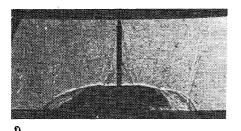
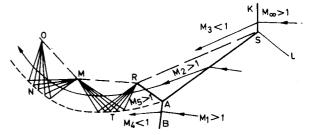
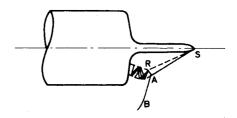


Fig. 2 Schlieren photographs of pulsation cycle.



a) - TYPE IV SHOCK FORMATION



b) - UNSTEADY FLOW COUNTERPART

Fig. 3 Similarity with Edney's IV shock formation.

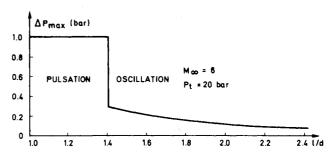


Fig. 4 Pressure fluctuation vs nondimensional spike length.

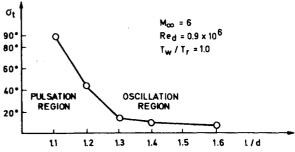


Fig. 5 Transition from pulsation to oscillation.

Strict similarity between a pulsating flow and an Edney type IV shock formation should not be expected because these flows cannot be described by a quasisteady analysis, since the Strouhal number based on the afterbody diameter is on the order 0.2. However, the appearance of the supersonic jet has been verified experimentally, as will be shown in the following section.

III. Verification of the Existence of the Supersonic Jet

A verification program was conducted in the von Kármán Institute High-Speed Laboratory. The major part of the experiments was carried out in a 15 cm blowdown wind tunnel, with running times of several minutes at $M_{\infty} = 6$.

The existence of the annular supersonic jet was first verified optically from Schlieren pictures of the flow about a spiked flat cylinder (Fig. 2). We note here that the jet is also visible in Fig. 32, p. 480 of Ref. 8. One may argue that what we see in these photographs is not a jet but the shear layer emanating from the shock intersection. But even if this is the case, its

existence indicates that the air which inflates the bubble is processed by the weak shock and does not come from the adjacent subsonic region. Besides, the pressure measurements reported in this section reveal the supersonic character of this jet or shear layer.

After the optical detection of the jet, we performed sublimation tests under flow conditions similar to those of Fig. 2. It was found that a dark ring was formed at the white surface of the cylinder, exactly where the annular supersonic jet impinges.

The dynamic characteristics of the pulsating flow were also detected by the pressure measurements. The measured amplitude of the pressure fluctuation at the face of the afterbody at r/R = 0.25 vs the spike length is shown in Fig. 4. The flow conditions were $M_{\infty} = 6$ and $p_t = 20$ bar. We observe that the amplitude of the fluctuation is independent of spike length for the pulsation mode and equal to $\Delta p = 1$ bar. If we normalize this value by the pressure p_3 behind the strong normal shock AB, we find that $\Delta p/p_3 = 2.0$. This simply shows that the fluctuation of the pressure in the separation region is greater than the pressure behind the strong shock and, consequently, that the air which fills the dead air region could not have come from the adjacent subsonic region but actually comes through the conical shock wave and is compressed at the impingement region.

IV. Indirect Evidence

According to the mechanism of the pulsation described in Sec. II, the rapid inflation of the separation bubble during the impulsive start of the flow co-occurs with the movement of the shock intersection away from the body. It is obvious that if in this phase the foreshock covers the afterbody before a sufficient quantity of high-pressure air is trapped in the bubble, then the explosive expansion of the bubble will not occur and consequently the pulsation mode will not be established.

In order to prove this hypothesis and thus provide indirect evidence for the validity of the proposed mechanism, we have studied experimentally the parameters which affect the development of the initial phase of the flow, i.e., the volume of the axisymmetric cavity and the inviscid position of the shock intersection.

Effect of the Volume of the Axisymmetric Cavity

If our hypothesis is valid, by increasing the volume of the axisymmetric cavity a limit must be reached above which the pulsation mode will not appear. One way of increasing the volume is to increase the length of the forebody. The effect of this parameter is shown in Fig. 4 for the case of the spiked cylinder. It is observed that indeed there exists a limit (1/d>1.4) above which the pulsation turns into oscillation.

As additional proof of the effect of the volume we mention a special test performed by Loll⁹ at VKI. Varying the dead-air region volume of a spiked flat cylinder by connecting it through a concentric hole with the inner part of the (hollow) cylinder, Loll found that for sufficient cylinder volume the flow became steady instead of pulsating.

Also a review of the experimental studies shows that if a concave body is in sufficient incidence, the pulsation mode does not appear. This happens because the high-pressure air escapes leeward.

Effect of the Position of the Shock Intersection

According to our hypothesis the nearer the shock intersection lies to the surface and to the shoulder of the afterbody, the faster it will be pushed radially outward and the smaller the quantity of the trapped air will be, providing fewer possibilities for the occurrence of pulsation. The effect of this parameter was studied by using flat-ended cylinders equipped with spikes of constant diameter and conical tips of variable angle. Testing step by step we measured the spike length for which the pulsation turns into oscillation. The results are

plotted in Fig. 5 and confirm the hypothesis of the importance of the initial position of the shock intersection in determining the type of the flow.

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Asymptotic Properties of the Zarf

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THE Orr-Sommerfeld equation for a plane parallel flow

$$(u(z),0,0) \tag{1}$$

is

$$\psi''''' - 2(\alpha^2 + \beta^2)\psi'' + (\alpha^2 + \beta^2)^2\psi$$

$$= iR\{(\alpha u - \omega)[\psi'' - (\alpha^2 + \beta^2)\psi] - \alpha u''\psi\}$$
 (2)

together with the boundary conditions $\psi(0) = \psi'(0) = 0$ and $\psi \to \infty$ as $z \to \infty$ is obtained by assuming that small disturbances to the basic flow can exist in the form

$$\exp\left[i\left(\alpha x + \beta y - \omega t\right)\right]\psi(z) \tag{3}$$

where R is a suitably defined local Reynolds number. This

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